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CASE STUDIES OF LEARNING MATHEMATICAL EXPRESSIONS BY THE STUDENTS OPTING HUMANITIES AT HIGHER SECONDARY STAGE

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Abstract

In spite of the ten years of study of mathematics, many of the learners'struggle with the basic operations and needs appropriate help to pursue mathematics further. The paper attempts to look into the mathematical abilities of the higher secondary stage students opting humanities group and to disclose the fundamental reasons of students' poor achievement through analysis of the errors they commit while solving the problems. The study results show that there were lacunae's in the learning of mathematics and most of students' errors occurred at comprehension and process skill level.

Key Words: Case Study, Learning Mathematical Expression, Humanities

Introduction

The significant role of mathematics in developing human thinking and in analyzing real life problems have given it the prominence to be usedeverywhere in the world, for many centuries as a universal language (Jha, 2012 [6]). Mathematics definition as a 'Science of order' (Whitehead, 1929 [15]) points on to its features- generalisation, simplification, representation, communication and application and determines its wide applications in economics, social studies and other fields.

It seems to be a fact of life that whilst a few prosper in mathematics a much greater number find mathematics difficult. Most of them who find mathematics difficult opt out of the subject and confines to arts or humanities courses. This option is available to them on completion of secondary education (class X). Till then they are forced to learn mathematics as a compulsory subject. During their ten years of study they come through all the basics of the subject but still, many of them struggle later and needs appropriate help to be able to pursue mathematics further.

The National Achievement Survey conducted by NCERT for class VIII shows that students committed errors on items based on mensuration, ratio and proportion. About 83

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percent of the surveyed students were not able to use laws of exponents. About 78 percent were not able to solve one variable linear equation in contextual problem involving multiplication and division of rational numbers (NCERT 2014 [11]). The Policy Research Working Paper of World Bank stated that the average learning levels are so low in India, that the typical child will leave primary school without knowing how to read or perform elementary mathematical operations. The absolute achievement, as measured by the percent correct score, was found to be low compared to curricular standards. The study results showed that a significant fraction of children have not mastered the content categories expected for their grade. The item-by-item comparison suggests that Indian children are performing significantly below the international average (Das, Jishnu & Tristan Zajonc, 2008 [4].

Background of the Study

The economics textbooks of class XII, *Introductory Microeconomics* and *Introductory Macroeconomics*, prepared by NCERT, on the basis of the recommendation of NCF 2005 have incorporated the recent developments in the theory aided by mathematical treatment of the issues involved. The mathematical tools with which school students are already familiar, such as algebra and solutions of simultaneous equation, have been attempted in the textbooks. This would enable them to approach and analyse with more realism the economic problems that they will confront in their day-to-day life.

Through different forums of interaction with teachers, it was learnt the students faced difficultieswhile dealing with the new textbooks and the most apparent being understanding the mathematical expressions used to explain economic theories mostly concerning the students of humanities stream. A research study was undertaken with the primary task of identifying difficulties encountered by students so that it may provide some suggestions for required changes in development of curricular materials. The study focused on diagnosing how much of knowledge students have in the basic mathematics, which will be a pointer towards the difficulties that might arise when they have to apply in learning economic theories.

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Objective

- To diagnose in a systematic way the difficulties faced by students while introducing mathematical language to communicate economic theories and to discuss the possible explanations for the problems.

Methodology

At the higher secondary stage economics is taken up by heterogeneous groups of students who branch into science stream, commerce stream, humanities stream and also vocational courses. Therefore, the ability range of students learning the subject represents a broader ability band. With regard to the specific problem raised in understanding the mathematical expressions of economic theories- it mostly concerns only the students of humanities stream. Here we are concerned with the performance of students belonging to the humanities stream who find difficulty with the mathematics that is a pre-requisite to learn the introductory economic theory course at higher secondary level. Students opting for humanities face a variety of challenges including low level of confidence with respect to mathematical ability, limited fluency in algebraic notations, limited understanding of the usefulness of mathematical principles in economic argument and difficulties in recognizing the mathematical representation of a problem within economic theory.

Sample of the Study

The study was conducted in two schools using economics textbooks prepared by NCERT which uses mathematical expressions to explain the economic theories.Students of two different schools were chosen as samples so as to ensure incorporation of all sorts of difficulties and qualitative differences in thinking process. As the teachers had expressed their concern for difficulties faced by humanities students while handling the mathematics due to lack of aptitude, negative orientations etc., the sample of the study included students who had opted for economics in the humanities group without mathematics as an elective subject.

Test Paper to investigate the mathematical skills of students

A preliminary mathematics diagnostic investigation was given to students of five different schools, using questions requiring an objective response which were designed to provoke

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possible misunderstandings and errors. For each of the questions on the diagnostic test, the proportion of correct, wrong and not attempted responses were calculated.

Data Analysis

The test papers of students on the basic math skills were evaluated and each question was analysed separately. The test results have been clubbed together under different heads depending on the type of mathematics involved. This helped in identifying the major difficulty areas in basic mathematics. The types of errors were analysed to work on its probable causes.

Results and Discussion

Apart from diagnosing difficulties, the nature, extent and causes of errors committed by the students were examined in the data analysis. While examining the mathematical skills of the students it was found that even though they were apprehensive regarding the mathematical operations to be carried out most of them fared well in certain type of question like solving of equation, identifying inequality signs and summation of equation. 77 percent of students were able to solve the equations correctly, 71.14 percent were able to correctly identify the inequality signs and 63.41 percent could sum up the equations. But their ignorance regarding certain basic math skills were brought out by the low percentage of correct answers on question related to functions (34.14 percent), carrying out arithmetic operations (24.39 percent) and indices (24.39 percent). The lowest number of correct answers was in question related to identification of modules sign (7.32percent) [Table I].

Table I: Correct Answers based on the type of Questions

School Type of Questions*	Percentage of Correct Answers
Indices	24.39
Functions	34.14
Solving of equations	77.00
Arithmetic operations	24.39
Inequality sign	71.14
Modulus	7.32
Summation of equations	63.41

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The test paper was prepared with the objective of diagnosing the specific weaknesses of students with regard to the arithmetic and algebraic skills. Apart from diagnosing difficulties, the nature, extent and causes of errors committed by the students were examined in the data analysis. While examining the mathematical skills of the students it was found that even though they were apprehensive regarding the mathematical operations to be carried out most of them fared well in certain type of question like solving of equation, identifying inequality signs and summation of equation.

Table II: Comparison between the Schools (%)

School Type of questions	School 1	School 2
Indices	11.76	38.78
Functions	30.43	41.18
Solving of equations	89.44	64.70
Arithmetic Operations	11.76	34.78
Inequality sign	82.61	59.80
Modulus	8.69	5.88
Summation of equations	78.26	47.06

Comparing the test results of the two schools it is found that the pattern of correct answers among the different type of questions remains the same. In School 1 it was seen that, 89.44 percent could solve equations, 82.61 percent identify the inequality signs and78.26 percent could sum up the equations. In School 2 also the higher percentage of correct answers was with respect to solving of equations (64.70 percent), identifying the inequality signs (59.80 percent) and summing up of the equations (47.06 percent). Only about 8.69 percent and 5.88 percent of students from School 1 and School 2 were able to identify the modulus sign.

The percentage share of correct answers was comparatively high for School1 for all type of questions. Almost 80 percent of the correct answers with regard to the low scoring questions based on indices and arithmetic operations were from the School 1. In respect of questions based on summation of equations, solving of equations and identification of inequality signs the percentage share of School 1 was 69.23 percent, 65.16 percent and 65.14 percent respectively [Table III].

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Table III: School-Wise Share of Correct Answers (%)

School Type of questions	School 1	School 2	Total
Indices	80	20	100
Functions	50	50	100
Solving of equations	65.16	34.84	100
Arithmetic Operations	80	20	100
Inequality sign	65.14	34.86	100
Modulus	66.67	33.33	100
Summation of equations	69.23	30.77	100

Analysis of Errors

Detailed examination of the errors committed by the students help in identifying their underlying difficulties. The students may commit errors as they are unable to read a key word or symbol that prevented him/her from proceeding further which is classified as reading errors; read all the words in the question correctly but had not understood the overall meaning and thus unable to proceed further classified as comprehension error; unable to identify the operation, or series of operations known as transformation error ; process skill errors in which the student is able to identify the appropriate operation, or series of operations, but did not know the necessary measures to carry out these operations perfectly and the encoding errors in which the student worked out the solution to a problem, but could not express the solution in an acceptable written form (Newman, M. A. 1977 [12]).

Arithmetic operations

Students committed errors in carrying out the arithmetic operations. They didn't apply the rule of order of precedence commonly coined as BODMAS rule to carry out the operations instead carried out operations reading from left to right. They treated the arithmetic symbols as a sequence of operations to be carried out in the order that they are read. This indicates that the students either have forgotten the order of precedence of arithmetic operations which they have learnt in earlier classes or they have not learnt it at all. On enquiry with the students most of them seemed to be totally unaware of the rule but some of them recognized the rule when the term 'BODMAS' was used. Only about 25 percent of students were able to chunk the symbols together as sub expression to be carried out according to given conventions.Research studies have shown that most students in the elementary grades are not

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aware of the underlying structure of arithmetic expressions. They do not understand for example, that 683 – 297 + 235 and 235 + 683 – 297 are equal without calculating (Kieran, 1989 [7], Chaiklin and Lesgold, 1984 [2]).

The Modules sign

Students often face difficulty in understanding the modules sign (Chiarugi et al, 1990 [3]). Only 7.32 percent could come up with correct answers. On enquiry it was learnt that they were unaware of such a symbol. Even those who had given correct answers had actually guessed it.

Identification of the inequality sign

A set of questions were posed in the form of true or false so as to find out whether the students had clear idea of what the signs represented. To avoid guessing of answers, group of questions were put forth. About 71.14 percent of students responded correctly to the questions. The percentage of students who could identify correctly the inequality signs of all the questions came down to 62.50 percent. The signs were not interpreted correctly and many of them seemed to be confused. Students experience difficulty in identifying the symbols and differentiating them.

Functions

Even though the question posed in function was rather very simple one, majority of the student responded in a manner that it can be clearly concluded that they did not have any idea of what a function is! Here the question was to find f (2) when f(x) = x-4/x+2. All they had to do was substitution of value of x but 65 percent of the students were not able to do it.

Algebra

Different sets of questions were raised to trace out the difficulties that arise while solving algebraic sign equations. Around 23 percent were not able to solve the equations. There were questions which involved negative sign, where RHS had to be solved and also equations involving fractions. There was disparity in the percentage of correct answers given for different questions. So, in order to diagnose the difficulty of the students each question has to be analysed separately instead of clubbing together the different set of questions. [Table IV]

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About 50 percent of students have difficulty in solving equations that involved negative numbers and which had variable on the RHS. Students face difficulty in solving equations that involve fractions. Around 33 percent had difficulty in solving equations involving fractions. When it came to finding solutions for simple equations majority of them were able to do it correctly. On an average, 87 percent of the students could solve all the simple equations. The high percent of wrong answers with respect to equations involving negative numbers and fractions points on to the fact that students did not have clear understanding of carrying out arithmetic operations involving negative numbers and fractions. This led to occurrence of errors while solving equations. Solving of equations involves procedures like "collect together like terms", turn upside down and multiply, 'do the same thing to both sides', 'change sides, change sign etc. Students find it difficult to handle these 'ill' remembered procedures.

Table IV: Solving of Equations

Type of questions	School 1 (%)	School 2 (%)	Total (%)
1. Solving for x, just by changing sides	95.65	76.47	87.5
2. Solving simple equations	100	76.47	90
3. Solving simple equations	95.65	64.70	82.5
4. Solving simple equations	100	70.59	87.5
5. Solving simple equations	95.65	76.47	87.5
6. Solving equations involving fractions	86.95	41.18	67.5
7. Solving equations involving negative numbers	52.17	47.05	50.00

Researchers have attributed students' difficulties in algebra to the lack of understanding of the letter/ variable (Kuchemann, 1981 [8] ; Booth, 1984 [1]; MacGregor and Stacey, 1997 [10]) and algebraic expression. Students' also find difficulty in grasping the process-product duality inherent in algebraic expressions, that is, the fact that the expression stands for a number as well as for instructions to perform operations on the number or letter (Sfard, 1991 [13]; Tall, 1999 [14]).

Summation of equation: Even tough 78.26 percent of students of School 1 and 47.06 percent of students of School 2 could sum up the equations correctly a through look into the incorrect answers may possibly point onto the difficulty faced by students. In case of question

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which involved summing up of numerical, majority of students answered it correctly but when it came to summation of equation involving variable 35 percent could not manipulate it correctly. They did not know how variables could be summed up, which points on to the fact that they did not have any idea how equations where formed and variables were substituted. If they had, the error of $p+2p=2p^2$ would never be committed. They were not aware of the fact that an expression is seen as a process to be carried out (2p+6 is multiply p by 2 and then add 6) and not as an object which can be manipulated. About 18 percent of students did not attempt the question which involved summation of equation.

Indices: Errors that occurred in the question where law of indices had to be carried out, shows that majority of the students were either unaware or forgotten the laws of indices. Unless the students knew the principles, they were likely to commit error or leave it without attempting. Among those who attempted, around 60 percent of students gave incorrect answers.

Table V: Analysis of Errors

S. No.	Questions	Mathematics required	Types of Errors
1.	Find 4 - 2 × 3+ 10÷ 2	Order of precedence in carrying out arithmetic operations	Ignored the order of precedence rule and carried out operations as they felt like
2.	If 4x-10=2x+2, the value of x is…….	Solving of equation	Equated the equation to zero to find solution, ignored the negative sign
3.	If - 10= -2, the value of P is …….	Solving of equation, Finding out common denominator	Doesn't know how to work with algebraic fractions, solving when negative numbers are involved.
4.	If x= 4 and y= 7 then Ix-yI=……	Identify modulus symbol	Went wrong in finding the value while subtracting big number from small number, doesn't know the symbol of modulus
5.	If f(x)= thenf (2) =……	Functions and finding out f(x) by substituting value of x in the	Ignored negative signs, didn't understand the question, made careless mistakes

		given function	
6.	Given x=p + 10 and y= p+5 then x + y is ……	Summation of Equations	Lacks understanding about variables in equations and is confused between operations, multiplication and addition.
7.	(i) P>10=…….. (ii)P $\geq$10=…… (iii) P$\leq$10=…… (iv) P< 10= ……..	Identification of inequality signs	Ignorance of symbol, where confused and wrongly interpreted the statements
8.	(i) Q= 0:p>10……. (ii) Q=0:p$\geq$ 10…….	Identification of inequality signs	Ignorance of symbol, where confused and wrongly interpreted the statements
9.	Suppose supply of two firms is given as S_1 =p+15 and S_2 = 2p-10, where p is the price, what will be the total supply S_1 + S_2?	Summation of Equations	Could not understand the task description and started to solve instead of summing up, is confused between operations, multiplication and addition that they wrote p+2p as $2p^2$
10.	Solve a) 5x+10=20 b) x+2=6 c) 3x=6 d) X+3=6 e) -7=p+2p+5	Solving of equations	Doesn't know the procedure of zero pairs, procedure of shifting variables from RHS to LHS, ignoring negative signs

Conclusions

Attainment in mathematics is very much based on the mastery of fundamental skills. It is observed through the analysis of the mathematical abilities and errors committed by the humanities students at higher secondary stage that they lacked the basic mathematical skills. The study establishes that the students were apprehensive towards using mathematical expressions due to their lack of mathematical abilities. Analysis of errors revealed how the lack of clear understanding of the basic math skills resulted in committing in correct answers. The students didn't know the rule order of precedence while carrying out arithmetic operations, how to work when negative numbers and fractions are involved and were unable to correctly identify the inequality signs. Linchevski and Livneh, 1999 [9] study showed connections in students' understanding of arithmetic and algebraic expressions as students tend to make the same kind of errors in both places.

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Misconceptions of basic concepts that persevere from early learning experiences were seen to be the cause of lack of understanding of mathematical expressions used in economics. Evidences indicate that carrying out operations involving negative numbers and fractions are the most difficult areas of mathematical teaching and learning.

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